Fractional Exponents & Radicals

Thursday October 10, 2013

- In grade 9, you learned that for powers with variable bases and whole number exponents

$$a^m \cdot a^n = a^{m+n}$$

- We can extend this law to powers with fractional exponents

Example:

$$5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5^{\frac{1+1}{2}} = 5^1 = 5$$

and

$$\sqrt{5} \cdot \sqrt{5} = \sqrt{25} = 5$$

$5^{\frac{1}{2}}$ and $\sqrt{5}$ are equivalent expressions, that is $5^{\frac{1}{2}} = \sqrt{5}$

- Raising a number to the exponent ½ is equivalent to taking the square root of the number.

- Raising a number to the exponent ⅓ is equivalent to taking the cube root of the number

...and so on.

Powers with Rational Exponents with Numerator 1.

Powers with Rational Exponents

When n is a natural number and x is a rational number, $x^{\frac{1}{n}} = \sqrt[n]{x}$

Example: Evaluate each power without using a calculator

Hint: The denominator of the exponent is the index of the radical.

a) $27^{\frac{1}{3}}$

b) $(-64)^{\frac{1}{3}}$

c) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

- As we know, a fraction can be written as a decimal (either terminating or repeating).
- Therefore we can interpret powers with decimal exponents

Example:

How do we solve $32^{0.2}$ without using a calculator?

$$0.2 = \frac{1}{5} \text{ , so } \quad 32^{0.2} = 32^{\frac{1}{5}}$$

Powers with Rational Exponents with Numerator other than 1

Recall:

$$(a^m)^n = a^{mn}$$

We can extend this law to powers with fractional exponents with numerator other than 1 such as $8^{\frac{2}{3}}$.

Example:

We can write the exponent $\frac{2}{3}$ as $\frac{1}{3}\cdot 2$ or, as $2\cdot\frac{1}{3}$

So, $8^{\frac{2}{3}} = 8^{\frac{1}{3}\cdot 2}$

$$= \left(8^{\frac{1}{3}}\right)^2$$

$$= \left(\sqrt[3]{8}\right)^2$$

$$= (2)^2$$

$$= 4$$

The numerator of a fractional exponent represents a power and the **denominator** represents a **root**.

Powers with Rational Exponents

When m and n are natural numbers, and x is a rational number,

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

and

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

Example 1

a) Write $26^{\frac{2}{5}}$ in radical form in 2 ways.

b) Write $\sqrt{6^5}$ and $\left(\sqrt[4]{19}\right)^3$ in exponent form.

Example 2: Evaluate

a) $27^{\frac{4}{3}}$

b) $0.04^{\frac{3}{2}}$

Example 3: Application

Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass m kilograms. Estimate the brain mass of each animal.

a) a moose with a body mass of 512 kg

b) A cat with a body mass of 5kg

Assignment